MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2015/2016

DIM5068 – MATHEMATICAL TECHNIQUES 2

(RS)

2 JUNE 2016
2.30 p.m – 4.30 p.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **3** pages.
2. Attempt **ALL** questions.
3. Please write all your answers in the answer booklet provided.
4. Formulas are provided in the appendix section.

Please answer ALL questions and show the necessary working steps. Each question is 20 marks.

QUESTION 1

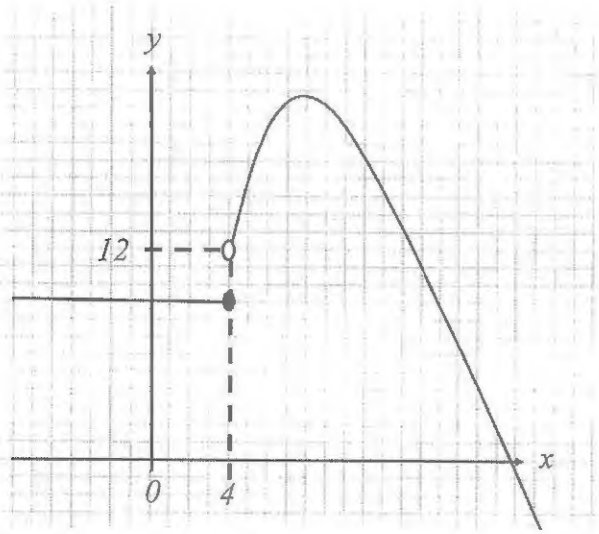
a. Find the values of m and n for the following equation. (4 marks)

$(7-6i)(-5+3i)-4=3m+ni$

b. Find the solution of the equation, $3x^3-6x^2+27x-54=0$. (6 marks)

c. The following graph shows the piecewise function of $f(x)$.

$$f(x)\begin{cases} 9 & if \quad x \le 4 \\ -x^2+12x-20 & if \quad x>4 \end{cases}$$



Find $\lim_{x\to 4} f(x)$. (3 marks)

d. Evaluate $\lim_{p\to\infty} \dfrac{5p^4-0.5p^3+6}{10p^4-1.8p^2}$. (4 marks)

e. If $\lim_{t\to 51} p(t)=k$, $\lim_{t\to 51} q(t)=-15$ and $3\lim_{t\to 51}[p(t).q(t)]=90$, show that value of k is -2. (3 marks)

[TOTAL 20 MARKS]

Continued...

QUESTION 2

a. Given $f(x)=7x^3-1$ and $g(x)=(4+x^2)^2$, answer the following questions:

i. If $y=f(x)\cdot g(x)$, find $\frac{dy}{dx}$ by using the **Product Rule**. (8 marks)

ii. If $y=\frac{f(x)}{g(x)}$, find $\frac{dy}{dx}$ by using the **Quotient Rule**. (5 marks)

b. Find the intervals of concavity and the inflection points of the function $f(x)=4x^3+3x^2+\frac{1}{2}$. (7 marks)

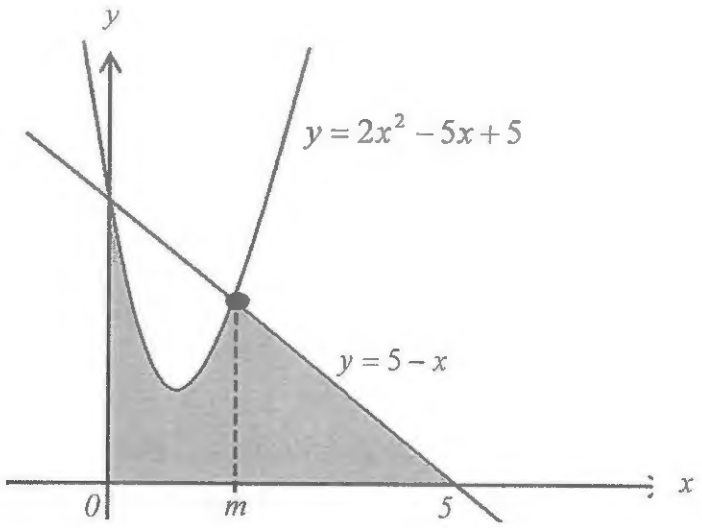
[TOTAL 20 MARKS]

QUESTION 3

a. Use **Substitution Rule** to find $\int(10x^3-1)(20x^4-8x+1)^9\,dx$. (6 marks)

b. Determine $\int 5xe^{-x}\,dx$ by using the **Integration by Parts**. (6 marks)

c. The diagram below shows the curve of $y=2x^2-5x+5$ and the straight line of $y=5-x$.



i. Show that the value of m is 2. (2 marks)

ii. Find the area of the shaded region. (6 marks)

[TOTAL 20 MARKS]

Continued...

QUESTION 4

a. Solve the differential equation $\frac{dp}{dq} = \frac{3q^2 + 4q - 1}{\sec^2 p}$ by using **separable method**.

(4 marks)

b. Solve the initial value problem $x\frac{dy}{dx} + 6y = 2x^3 - \frac{e^x}{x^5}$ given that $y(0) = 1$.

(11 marks)

c. Find the general solution for the differential equation $y'' - 5y' - 14y = 0$.

(5 marks)

[TOTAL 20 MARKS]

QUESTION 5

a. For the given vectors $\boldsymbol{w} = -2\boldsymbol{i} + 5\boldsymbol{j} + 3\boldsymbol{k}$, $\boldsymbol{v} = 11\boldsymbol{i} + 4\boldsymbol{k}$ and $\boldsymbol{p} = -\boldsymbol{i} + 33\boldsymbol{k}$,

i. find the dot product of $\boldsymbol{w}$ and $\boldsymbol{v}$. (2 marks)

ii. based on the result in part a.(i), determine whether $\boldsymbol{w}$ and $\boldsymbol{v}$ are orthogonal. (1 mark)

iii. find the cross product of $\boldsymbol{v}$ and $\boldsymbol{p}$. (4 marks)

iv. find $\boldsymbol{v} - 2\boldsymbol{p}$. (3 marks)

v. based on the result in part a.(iv), determine the magnitude of $\boldsymbol{v} - 2\boldsymbol{p}$. (2 marks)

b. Given that vectors $\boldsymbol{r} = \langle 3, 5, 1\rangle$, $\boldsymbol{m} = \langle 9, 1, 4\rangle$ and $\boldsymbol{n} = \langle 2, b, \frac{1}{3}\rangle$, find the value of b if $5\boldsymbol{r} - \boldsymbol{m} = 3\boldsymbol{n}$. (4 marks)

c. Find the equation of the plane through the point (22, 0, 1) and perpendicular to the vector <1, -8, 3>. (4 marks)

[TOTAL 20 MARKS]

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APPENDIX

Derivatives: $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Differentiation Rules

General Formulae

1. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
2. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$

Exponential and Logarithmic Functions

1. $\frac{d}{dx}(e^x) = e^x$
2. $\frac{d}{dx}(a^x) = a^x \ln a$
3. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
4. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Trigonometric Functions

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x$
3. $\frac{d}{dx}(\tan x) = \sec^2 x$
4. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
5. $\frac{d}{dx}(\sec x) = \sec x \tan x$
6. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Table of Integrals

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{du}{u} = \ln|u| + C$
4. $\int e^u du = e^u + C$
5. $\int \sin u\,du = -\cos u + C$
6. $\int \cos u\,du = \sin u + C$
7. $\int \sec^2 u\,du = \tan u + C$
8. $\int \csc^2 u\,du = -\cot u + C$
9. $\int \sec u \tan u\,du = \sec u + C$
10. $\int \csc u \cot u\,du = -\csc u + C$

Application of Integrals:

Areas between Curve, $A = \int_a^b [f(x) - g(x)]\,dx$

Differential Equations

Linear Differential Equations

$$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx, \text{ where } \mu = e^{\int p(x)dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Area for parallelogram *PQRS*

$= \left| \vec{PQ} \times \vec{PR} \right|$

Area for triangle PQR

$= \dfrac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$

Symmetric equation: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\cos\theta = \dfrac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}$